Lars Karlsson
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HKGAB4 - Lab 11-12

Linköping University
IDA
CogSci Line

Lab 11-12 : Quantification II & Fundamental Concepts Revisited

1 Goal

The goal of these labs is to continue practice with quantification. In particular, we focus on

- Translating to and from Natural Language Sentences.
- Informal and Formal Proof Theory for the Quantifiers, including mixed quantifiers.

In addition, as this is the last lab, we repeat some of the fundamental concepts of logic.

2 Preparation

The following should be done before the actual lab session:

- Read Chapter 6 in LFOL.
- The following LFOL exercises can be prepared ahead of time:

  LFOL, Chap. 6: Exercises 6-8, 18, 38-41, 45-48.

3 Lab Exercises

The following lab exercises are obligatory and should be handed in unless explicitly stated otherwise.

Exercise	**SaveAs (Apple)**	**SaveAs (IBM)**
6.1	Do not hand in.	
6.2, 6.3	World 6.x	6-x.wld
6.5	Do not hand in.	
6.6, 6.7, 6.8	World 6.x	6-x.wld
6.10	Do not hand in.	
6.11, 6.14, 6.15	World 6.x	6-x.wld
6.16	Sentences 6.16	6-16.sen
6.17	World 6.17	6-17.wld
6.18	Sentences 6.18	6-18.sen
6.38, 6.39, 6.40, 6.41	World 6.x	6-x.wld
6.50	World 6.50	6-50.wld
6.51	Sentences 6.51	6-51.sen

In addition, do the following exercises from LoFOL in Hyperproof. The problem files are in the Hyperproof folder LoFOL. To do the problem, simply click on the problem file. It will be loaded and the file name under which you should save the solution will automatically be set up. When you have finished the exercise, simply click on "Save" in the "Arkiv" menu. These exercises are obligatory and should be handed in.

Exercise	
LoFOL 6.45-48	-

4 Some fundamental concepts revisited

These last assignments will repeat some of the fundamental concepts in logic. To think about:

- What kinds of grammatical elements are there in first-order logic, and how can they be used?
- What does logical consequence and nonconsequence mean in terms of (a) models (worlds, perceivable circumstances) and (b) proofs?
- What does logical truth, satisfiability and falsity mean in terms of (a) models and (b) proofs?
- How can a problem of logical consequence ($P_1, \ldots, P_n \models Q$) be turned into a problem of logical truth?

The following exercises should be done individually. You may use Hyperproof, but then the solutions should be printed out on paper.

1. What (if anything) is wrong about each one of the following logical formulae?
   (a) $(a = b) \rightarrow (a \vee b)$
   (b) $\forall x, y[Owl(x) \rightarrow (Owl(y) = a)]$
   (c) $\exists x[House(x) \vee Hut(x)]$

2. Translate the following premises to first-order logic.
   Jor-El is the father of Superman.
   Anybody Kryptonese has superpowers.
   If one has superpowers, one can lift the Statue of Liberty.
   If your father is a Kryptonese, you are also a Kryptonese.
   Jor-El is a Kryptonese.
   Clark Kent is identical to Superman.
   Lois Lane is in love with Superman.
   Then show (formally) that each of the following statements is a consequence, or that it is not:

   (a) Lois Lane can lift the Statue of Liberty.
   (b) Lois Lane cannot lift the Statue of Liberty.
   (c) There is somebody who is in love with somebody who can lift the statue of liberty.
   (d) Lois Lane is in love with Clark Kent.

   If you think that there is something weird about (d), you are not alone (after all, Lois Lane doesn't know that Clark is Superman, or at least didn't use to know.) Many logicians have been working on this problem, and we can recommend chapter 15 in *Mathematical Methods for Linguistics* if you want to learn more.

3. An ornithologist once decided to write down his knowledge about birds in logic. He came up with the following set of axioms (premises).

$$\begin{array}{l} \forall x[Bird(x) \to Flies(x)] \\ \forall x[Penguin(x) \to \neg Flies(x)] \\ \forall x[Penguin(x) \to Bird(x)] \\ Penguin(Tweety) \end{array}$$

   Show (formally) for each one of the following statements that it is a consequence, or that it is not:

   (a) $Flies(Tweety)$.
   (b) $\neg Flies(Tweety)$.

   There is a problem with this set of axioms. What? Fix this problem in a reasonable way while modifying the axioms as little as possible. With the modified axioms, show consequence/nonconsequence for (a) and (b) again.

4. Show (formally) for each one of the following statements whether it is tautologically true, satisfiable or tautologically false.

   (a) $\forall y[Dog(y)] \to \forall x, y[Dog(x) \vee Cat(y)]$.
   (b) $\forall y[Dog(y)] \wedge \forall z[Cat(z)] \wedge \exists v[Cat(v) \to \neg Dog(v)]$.
   (c) $\forall y[Dog(y)] \to \forall x[Cat(x)]$.

   If you want to motivate your answer with a world (model), then you can decide yourself what objects there are in this world.

5. When we write down a number of logical formulae as premises, what do we mean, i.e. what is the point of doing that? If we start with a blank piece of paper and add a logical sentence, what happens?

6. During the course we have repeatedly translated declarative sentences in English to sentences in logic. The meaning of an English sentence is often ambiguous and dependent on who wrote it, the context it was written, and so on. In logic we have a precise notion of the meaning of a sentence. What happens to the meaning of the English sentence in the translation process? If a translated sentence turns out to be a logical truth, what does that say about the original English sentence? If it is a contradiction? If it is neither a logical truth, nor a contradiction?

7. Just one more to go. Either provide a convincing argument that for any set of premises $P_1, \ldots, P_n$ and any sentence Q, it is always the case that either $P_1, \ldots, P_n \models Q$ or $P_1, \ldots, P_n \models \neg Q$, or provide a convincing argument that this is not always the case.

   Based on this argument *and* a certain central theorem about the relation between truth and consequence, either show that it is the case or that it is not the case for any set of sentences $P_1, \ldots, P_n, Q$ that either $(P_1 \wedge \ldots \wedge P_n) \rightarrow Q$ is a tautological truth or $(P_1 \wedge \ldots \wedge P_n) \rightarrow \neg Q$ is.

8. Go out and take a walk in the sun! (You don't have to hand in a written solution for this exercise.)

5 Deadlines

Each student/group should hand in his/her/their diskette and each student his/her written solutions by

- Tuesday April 6, 1999.

We in turn will try to have the graded exercises back to you by

- Friday April 9, 1999.

Exercises handed in after this date can <u>at best</u> get a grade of *Godkänd*, providing they are done correctly.

6 Discrete Mathematics deadline

All your discrete mathematic assignments should also be handed in by

- Tuesday April 6, 1999.