Logic + Control: An example
or
SAT solver of Howe & King as a logic program

(File `./LIPIcs/29.pdf`)

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This file contains extra material, not intended to be shown within a short presentation. In particular, such are all the slides with their titles in parentheses.

Is there logic / "logic" in actual Logic Programming ?

To which extent LP is declarative/logical ?

How to reason about logic programs?

We present

a construction of a practical Prolog program

(SAT solver of Howe&King).

Most of the reasoning done at the declarative level

(formally)

abstracting from any operational semantics.

Plan

- Specification
- Proving correctness & completeness
- Logic programs 1, 2, 3
- Adding control
- Conclusions

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Preliminaries

Definite programs.

To describe relations to be defined by program predicates:

Specification – a Herbrand interpretation S.

Specified atom – a $p(t_1, \ldots, t_n) \in S$.

Representation of propositional formulae

for a SAT solver [Howe&King]

Literals	x	$\neg x$
as pairs	`true-X`	`false-X`

CNF formulae $(\ldots \wedge (\ldots \vee \mathit{Literal}_{ij} \vee \ldots) \wedge \ldots)$

as lists of lists $[\ldots,\ [\ldots,\ \mathit{Pair}_{ij},\ \ldots],\ \ldots]$

CNF formula $[f_1, \ldots, f_n]$ is satisfiable iff
it has an instance $[f_1\theta, \ldots, f_n\theta]$ where $\forall_i$
$$f_i\theta \in L_1^0 = \{\, [t_1\text{-}u_1, \ldots, u\text{-}u, \ldots, t_n\text{-}u_n] \in \mathcal{H} \,\}\,.$$

CNF formula f is satisfiable iff
some $f\theta$ is in $L_2^0 = \{\, [f_1\theta, \ldots, f_n\theta] \mid \text{as above} \,\}$.

A program defining L_2^0 is a SAT solver.

Specifying a SAT solver

So apparently

a SAT solver should compute L_2^0.

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a SAT solver should compute L_2^0.

Computing exact L_2^0 unnecessary.

E.g. nobody uses `append/3` defining the list appending relation exactly!

☞ Common in LP: relations to be computed known approximately.

Specifying a SAT solver

So

a SAT solver ~~should~~ may compute L_2^0.

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Specifying a SAT solver

So

a SAT solver ~~should~~ may compute L_2^0.

Also
it may compute a certain $L_2 \supseteq L_2^0$.

$$L_2 = \{\, s \in \mathcal{H} \mid \text{ if } s \text{ is a list of lists of pairs then } s \in L_2^0 \,\}\,.$$

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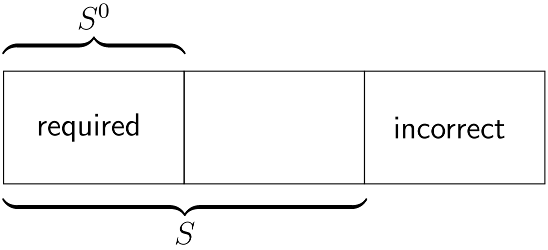
$$L_2 = \{\, s \in \mathcal{H} \mid \text{ if } s \text{ is a list of lists of pairs then } s \in L_2^0 \,\}\,.$$

Any set $L_2^0 \subseteq L_2' \subseteq L_2$ will do:

a CNF formula f is satisfiable iff some $f\theta$ is in L_2'.

☞ Common in LP: relations to be computed known approximately.

Approximate specifications

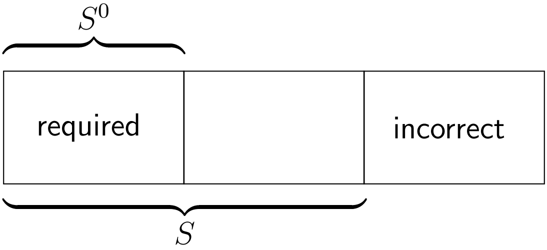


Approximate specification – (S^0, S), where $S^0 \subseteq S$.

$\uparrow$ for completeness $\uparrow$ for correctness

Intention: $S^0 \subseteq M_P \subseteq S$. S^0 – what has to be computed.
S – what may be computed.

Approximate specifications



Approximate specification for SAT solver: (S_1^0, S_1),

states that predicate sat_cnf defines a set L_2': $L_2^0 \subseteq L_2' \subseteq L_2$.

[Details $\rightsquigarrow$ the paper]

(Details – 1st specification for SAT solver)

Specification: (S_1^0, S_1) with the specified atoms

S_1^0:

$$\begin{array}{lll} sat\_cnf(t), & \text{where} & t \in L_2^0, \\ sat\_cl(s), & & s \in L_1^0, \\ x = x, & & x \in \mathcal{H} \end{array}$$

S_1:

$$\begin{array}{lll} sat\_cnf(t), & \text{where} & t \in L_2, \\ sat\_cl(s), & & s \in L_1, \\ x = x, & & x \in \mathcal{H} \end{array}$$

$$L_1^0 = \{ [t_1\text{-}u_1, \ldots, u\text{-}u, \ldots, t_n\text{-}u_n] \in \mathcal{H} \},$$

$$L_2^0 = \{ [s_1, \ldots, s_n] \mid s_1, \ldots, s_n \in L_1^0 \},$$

$$L_1 = \{ t \in \mathcal{H} \mid \text{ if } t \text{ is a list of pairs then } t \in L_1^0 \},$$

$$L_2 = \{ s \in \mathcal{H} \mid \text{ if } s \text{ is a list of lists of pairs then } s \in L_2^0 \}.$$

Correctness & completeness of programs

Correctness (imperative programming)

$$\swarrow \quad \searrow$$

Correctness $M_P \subseteq S$ Completeness $S \subseteq M_P$ (logic programming)

Completeness:

Everything required by the spec. is computed.

Correctness:

Everything computed is compatible with the spec.

P semi-complete w.r.t. S

$= P$ complete for terminating queries (under *some* selection rule).

[Details ⇝ the paper]

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Correctness & completeness, sufficient conditions

Th. (Clark 1979): P **correct** w.r.t. S when
for each $(H \leftarrow B) \in ground(P)$, $B \subseteq S \Rightarrow H \in S$.

(Out of correct atoms, the clauses produce only correct atoms.)

Th.: P **semi-complete** w.r.t. S when
for each $H \in S$,
exists $(H \leftarrow B) \in ground(P)$ where $B \subseteq S$.

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Semi-complete + terminating $\Rightarrow$ complete.

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SAT solver 1

P_1:

$sat\_cnf([\,]).$

$sat\_cnf([Clause|Clauses]) \leftarrow sat\_cl(Clause),\ sat\_cnf(Clauses).$

$sat\_cl([Pol\text{-}Var|Pairs]) \leftarrow Pol = Var.$

$sat\_cl([H|Pairs]) \leftarrow sat\_cl(Pairs).$

Can be constructed

guided by the sufficient conditions above, and specification (S_1^0, S_1).

Correct w.r.t. S_1, complete w.r.t. S_1^0. [Details $\rightsquigarrow$ the paper]

Inefficient backtracking search.

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Inefficient backtracking search.

(Towards better efficiency)

Idea: Watch two variables of each clause.
Delay $Pol = Var$ in $sat\_cl([Pol\text{-}Var|Pairs]) \leftarrow Pol{=}Var$
until Var watched and bound.

New predicates – another representations of clauses

E.g. (v_1, p_1, v_2, p_2, s) for $[p_1\text{-}v_1, p_2\text{-}v_2|s]$.

To block on v_1, v_2

Specification (S_1^0, S_1) extended $\rightsquigarrow$ (S_2^0, S_2).

Guided by the sufficient conditions for correctness & completeness
a logic program P_2 built,

correct & complete w.r.t. the new specification.

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(Towards efficiency. Details: the new spec.)

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New predicates. Specification: S_1^0 (resp. S_1) extended by atoms

$sat\_cl3(s, v, p)$, where $[p\text{-}v|s] \in L_1^0$ (resp. $\in L_1$),
$sat\_cl5(v_1, p_1, v_2, p_2, s)$,
$sat\_cl5a(v_1, p_1, v_2, p_2, s)$, $[p_1\text{-}v_1, p_2\text{-}v_2|s] \in L_1^0$ (resp. $\in L_1$).
Already in S_1^0 (S_1):
$sat\_cl(s)$ $s \in L_1^0$ (resp. $\in L_1$).

Intention: v_1, v_2 – the watched variables
`:-block sat_cl5(-,?,-,?,?)`
sat_cl5a called with v_1 bound

(Towards efficiency, final logic program)

P_2 may flounder (under the intended delays).

To avoid floundering – new predicates, new specification.

Initial queries $sat(f, l)$

$\uparrow$

Variables in f

Spec. requires l to be a list of $true/false$

Guided by the sufficient conditions for correctness & completeness
a logic program $P_3 \supseteq P_2$, correct & complete.

[Details ⇝ the paper]

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Towards better efficiency – brief

To prepare the intended control – new predicates.

E.g. another data representation, like
(v_1, p_1, v_2, p_2, s) for $[p_1\text{-}v_1, p_2\text{-}v_2|s]$,
to block on v_1, v_2.

Specification (S_1^0, S_1) extended $\rightsquigarrow$ (S_3^0, S_3).

Guided by the sufficient conditions for correctness & completeness
a logic program P_3 built
correct & complete w.r.t. the new specification.

Adding control to P_3

- Delays – modifying the selection rule

  `:-block sat_cl5(-,?,-,?,?)`

- Two cases of pruning SLD-trees.

  Skipping a rule of P_3; implemented by `(...->...;...)`.

  Completeness preserved.

  Case 1 – proof [technical report].

  Case 2 – informal justification

Result: Prolog program [Howe&King] of 22 lines / 12 rules.
Implements DPLL with watched literals and unit propagation.
(partly)

(Adding control, details)

Delays – modifying the selection rule

```
:-block sat_cl5(-,?,-,?,?)
```

Pruning 1. Choosing one of two clauses dynamically.
Completeness preserved. [Proof → tech. report]

$$sat\_cl5(Var1,\ldots,Var2,\ldots) \leftarrow sat\_cl5a(Var1,\ldots,Var2,\ldots).$$
$$sat\_cl5(Var1,\ldots,Var2,\ldots) \leftarrow sat\_cl5a(Var2,\ldots,Var1,\ldots).$$

$$\rightsquigarrow$$

$$\begin{array}{l} sat\_cl5(Var1,\ldots,Var2,\ldots) \leftarrow \\ \quad nonvar(Var1) \rightarrow sat\_cl5a(Var1,\ldots,Var2,\ldots) \\ \quad\quad ;\ sat\_cl5a(Var2,\ldots,Var1,\ldots). \end{array}$$

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(Adding control, details 2)

Pruning 2. Removing a redundant part of SLD-tree.

(Do not work on a clause which is already true.)

Completeness preserved, informal justification.

$$
\begin{array}{l}
sat\_cl5a(Var1, Pol1, \_, \_, \_) \leftarrow Var1 = Pol1. \\
sat\_cl5a(\_, \_, Var2, Pol2, Pairs) \leftarrow sat\_cl3(Pairs, Var2, Pol2).
\end{array}
$$

$$\rightsquigarrow$$

$$
\begin{array}{l}
sat\_cl5a(Var1, Pol1, Var2, Pol2, Pairs) \leftarrow \\
\qquad Var1 = Pol1 \ \rightarrow \ true; \quad sat\_cl3(Pairs, Var2, Pol2).
\end{array}
$$

Conclusions, proving correctness & completeness

Proving correctness.
Method of [Clark'79] simpler not weaker than that of Bossi&Cocco [Apt'97].

☹ Neglected.

Proving completeness. Seldom considered. (E.g. not in [Apt'97].)
Our method: new notion of semi-completeness,
semi-completeness + termination ⇒ completeness.

Both methods

- ☺ simple, natural, declarative (but termination),
- ☺ correspond to common-sense reasoning about programs,
- ☺ applicable in practice (maybe informally).

Ex.: An error in P_1 (first version) found & located by a failed proof attempt.

Methods for programs with negation: [Drabent,Miłkowska'05]

Conclusions, approximate specifications

➔ Approximate spec's crucial for formal / precise reasoning about programs.

Exact relations (defined by programs) often not known,
not easy to understand.

Ex.: Which set is defined by $sat\_cl/1$ in P_1? In P_2, P_3?
Misunderstood by the author (first report) and some reviewers.

➔ Approximate spec's useful for declarative diagnosis (DD).
Trouble: DD requires exact specifications.
Ex. Is $append([a], b, [a|b])$ correct?

Approximate spec's should be used:

Diagnosing incorrectness / incompleteness – specification for correctness / completeness

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Conclusions, approximate specifications 2

Transformational approaches seem inapplicable
to our example $P_1 \rightsquigarrow P_3$,

as the same predicates define different sets in P_1, P_3.
have the same approximate specification

Interpretations as specifications
– "existential specifications" inexpressible. :(

Ex.: We could not state that
for each satisfiable f some true instance $f\theta$ is computed.
We required all true instances.

Solution(?): Use *theories* as specifications.

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Conclusions, declarative programming

Most of reasoning can be done
at declarative level / pure logic programs.

Abstracting from operational semantics,
thinking in terms of relations;
formally.

Separation "logic" – "control" works:

Reasoning related to operational semantics / efficiency
independent from that related to correctness & semi-completeness.

But: Pruning may spoil completeness.

Conclusions, . . .

Claim: The presented approach can be used in practice,
maybe informally,
in programming and in teaching.

LP is not declarative unless
we have/use declarative means of reasoning about programs.

Conclusions, summary

- Approximate specifications crucial

  Approximate spec's useful for declarative diagnosis

- Simple methods for proving correctness & completeness

  declarative (but termination)
  applicable in practice

- Most of reasoning can be done at declarative level

  (pure logic programs)

  Declarative properties
  Operational properties – reasoning independent

- Claim: Approach practically applicable maybe informally,
  in programming and in teaching.

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